iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

The ultimate solution to the quantum Battle of the Sexes game

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2009 J. Phys. A: Math. Theor. 42 365305

(http://iopscience.iop.org/1751-8121/42/36/365305)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.155
The article was downloaded on 03/06/2010 at 08:08

Please note that terms and conditions apply.

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 365305 (7pp) doi:10.1088/1751-8113/42/36/365305

The ultimate solution to the quantum Battle of the Sexes game

Piotr Frąckiewicz

Institute of Mathematics of the Polish Academy of Sciences, 00-956 Warsaw, Poland

E-mail: P.Frackiewicz@impan.gov.pl

Received 3 June 2009, in final form 27 July 2009
Published 24 August 2009
Online at stacks.iop.org/JPhysA/42/365305

Abstract
We present the unique solution to the quantum Battle of the Sexes game. We show the best result to be achieved when the game is played according to Marinatto and Weber's scheme. The result which we put forward does not surrender the criticism of previous works on the same topic.

PACS numbers: 03.67.−a, 02.50.Le

1. Introduction

Theory of games concerns the description of conflict situations between two or more individuals, usually called players.

For about the last 10 years, next to the classical theory of games, a new field of investigation—quantum games [1]—has been developing. It represents an extension of traditional theory of games into the field of quantum mechanics (quantum information). In quantum games, players have access to strategies which are not encountered in the 'macroscopic world'. This phenomenon leads to new and interesting results which may be attained by players equipped with quantum strategies [2–4].

1.1. Battle of the Sexes game

The Battle of the Sexes is a static two-player game of nonzero sum whose matrix representation is as follows:

$$\Gamma : \begin{matrix} & q=1 & q=0 \\ \begin{matrix} p=1 \\ p=0 \end{matrix} & \multicolumn{2}{c}{\begin{bmatrix} (\alpha,\beta) & (\gamma,\gamma) \\ (\gamma,\gamma) & (\beta,\alpha) \end{bmatrix}} \end{matrix}, \qquad \text{where} \quad \alpha > \beta > \gamma. \tag{1}$$

Characteristic for the Battle of the Sexes game are three Nash equilibria: one is found in mixed strategies, and the other two in pure strategies. The first player prefers equilibrium (1, 1) which yields him the payoff α. In turn, the second player, in order to get the payoff α, prefers (0, 0). The problem of opposing expectations of the two players constitutes a definite dilemma. The players, following their preferences, may play a strategy profile (1, 0) that gives them the payoff γ—the worst payoff in the game.

1.2. The model of the quantum game

The quantum model of a two-player static game (the game in which each player chooses their strategy once, and the choices of all players are made simultaneously) is a family $(\mathcal{H}, \rho_{\text{in}}, U_A, U_B, \succeq_A, \succeq_B)$ [3]. In such a model, $\mathcal{H}$ is the underlying Hilbert space of the physical system used to play a game, and ρ_{in} is the initial state of this system. Sets of strategies of two players are sets U_A and U_B of unitary operators by which players can act on ρ_{in}. The symbols $\succeq_A$ and $\succeq_B$ mean the preference relation for the first and the second player, respectively, which can be replaced by the payoff function. The first scheme for playing a quantum 2×2 game in which both players have access to 'quantum' strategies appeared in [5]. In this model, Hilbert space $\mathcal{H}$ is defined as $\mathrm{C}^2 \otimes \mathrm{C}^2$. The players apply unitary operators acting on C^2 which depend on two parameters. The initial state ρ_{in} is taken to be a maximally entangled state of two qubits. Marinatto and Weber [6] introduced a new scheme for quantizing 2×2 games. In contrast to the scheme proposed in [5], they restricted players' actions to applying an identity operator I or a Pauli operator σ_x, or any probabilistic mixture of I and σ_x. This limitation of unitary operators can lead to the situation in which the players are even unable to state whether they play a game in the classical or in the quantum form [7]. For this reason, Marinatto and Weber's scheme seems to be the more natural way for quantizing games. In the following section, we give a precise description of this scheme.

2. General Marinatto–Weber scheme

In the Marinatto–Weber scheme of playing 2×2 quantum games, a space state of a game is the $2 \otimes 2$-dimensional complex Hilbert space with a base $(|00\rangle, |01\rangle, |10\rangle, |11\rangle)$. The initial state of a game is $|\psi_{\text{in}}\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ and $I, C = \sigma_x$ are, respectively, identity and bit-flip operators. Players are able to manipulate the initial state $|\psi_{\text{in}}\rangle$ by acting by I or C on the first and the second entry in the ket $|\cdot\cdot\rangle$, respectively. According to the idea of mixed strategies, they can also apply, respectively, $pI + (1-p)C$, $qI + (1-q)C$, where $0 \leqslant p, q \leqslant 1$. If the language of density matrices is used, then $\rho_{\text{in}} = |\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|$ and the final state of the game is as follows:

$$\begin{aligned}\rho_{\text{fin}} = {} & pqI_1 \otimes I_2\rho_{\text{in}}I_1 \otimes I_2 + p(1-q)I_1 \otimes C_2\rho_{\text{in}}I_1 \otimes C_2 \\ & + (1-p)qC_1 \otimes I_2\rho_{\text{in}}C_1 \otimes I_2 + (1-p)(1-q)C_1 \otimes C_2\rho_{\text{in}}C_1 \otimes C_2. \end{aligned} \tag{2}$$

When the original classical game is defined by a bi-matrix,

$$\Lambda : \begin{matrix} & q=1 & q=0 \\ p=1 & (x_{11}, y_{11}) & (x_{12}, y_{12}) \\ p=0 & (x_{21}, y_{21}) & (x_{22}, y_{22}) \end{matrix}, \tag{3}$$

the payoff operators are

$$P_A = x_{11}|00\rangle\langle 00| + x_{12}|01\rangle\langle 01| + x_{21}|10\rangle\langle 10| + x_{22}|11\rangle\langle 11|, \tag{4}$$

$$P_B = y_{11}|00\rangle\langle 00| + y_{12}|01\rangle\langle 01| + y_{21}|10\rangle\langle 10| + y_{22}|11\rangle\langle 11|. \tag{5}$$

The payoff functions π_A and π_B can then be obtained as mean values of the above operators:

$$\pi_A = \mathrm{Tr}\{P_A\rho_{\text{fin}}\}, \qquad \pi_B = \mathrm{Tr}\{P_B\rho_{\text{fin}}\}. \tag{6}$$

After applying the procedure discussed above, the quantum equivalent of the classical game Λ (3) is characterized by a two-dimensional bi-matrix Λ_Q, the elements of which are specified as a product of two matrices:

$$\pi(i,j) = (|a_{i\oplus_2 1, j\oplus_2 1}|^2 \quad |a_{i\oplus_2 1, j}|^2 \quad |a_{i, j\oplus_2 1}|^2 \quad |a_{ij}|^2)(X,Y), \tag{7}$$

where $\pi(i,j) = (\pi_A(i,j), \pi_B(i,j))$, $i,j \in \{0,1\}$, $\oplus_2$ means addition modulo 2 and $(X,Y) = ((x_{11}, y_{11}) \quad (x_{12}, y_{12}) \quad (x_{21}, y_{21}) \quad (x_{22}, y_{22}))^T$.

In the special case when $|\psi_{\text{in}}\rangle = |00\rangle$ an equality $\Lambda = \Lambda_Q$ occurs.

3. Various attempts at solving the dilemma of the quantum Battle of the Sexes game

The history of efforts put into the quantum solution to the dilemma that unavoidably occurs in the classical Battle of the Sexes began in [6], where a scheme of playing quantum games alternative to the scheme proposed in [5] was published. Marinatto and Weber showed that the players who have access to quantum strategies may gain the same payoff in every equilibrium. If the initial state of the game is $|\psi_{\text{in}}\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ then instead of (α, β) or (β, α), respectively, they obtain $((\alpha+\beta)/2, (\alpha+\beta)/2))$ for strategy profiles $(1,1)$ and $(0,0)$. Equalization of payoffs for players obtained in both equilibria certainly eliminates the difference between preferences of the players but, as Benjamin [8] correctly stated, the dilemma still exists. In spite of the fact that both players prefer two equilibria to the same extent, there is still a possibility that because of lack of communication between both players they may obtain the worst payoff γ, which happens when they play combinations of strategies $(1,0)$ or $(0,1)$.

Further improvement in solving the dilemma of the Battle of the Sexes game (1) was presented by Nawaz and Toor in [9]. They improved the results of [6] considering the quantum game Battle of the Sexes that begins with the initial state $|\psi_{\text{in}}\rangle = (\sqrt{5}\mathrm{e}^{\mathrm{i}\phi_1}|00\rangle + \sqrt{5}\mathrm{e}^{\mathrm{i}\phi_2}|01\rangle + \mathrm{e}^{\mathrm{i}\phi_3}|10\rangle + \sqrt{5}\mathrm{e}^{\mathrm{i}\phi_4}|11\rangle)/4$ and showing that it is equivalent to the classical game characterized by the following payoff bi-matrix:

$$\Gamma_{\text{NT}} = \frac{1}{16}\begin{bmatrix} (\alpha', \alpha') & (\beta', \gamma') \\ (\gamma', \beta') & (\alpha', \alpha') \end{bmatrix}, \tag{8}$$

where $\alpha' = 5\alpha + 5\beta + 6\gamma$, $\beta' = 5\alpha + \beta + 10\gamma$, $\gamma' = \alpha + 5\beta + 10\gamma$. Then they argued that each player should choose their first strategy. It can easily be observed that [9] improved results of [6]. For any α, β, γ where $\alpha > \beta > \gamma$, it is better for both players to play 'Nawaz and Toor's game' than 'Marinatto and Weber's game': in [6], if players choose their strategies 1 or 0 at random, they gain with equal probability $(\alpha+\beta)/2$ and γ, which gives them the expected value $(\alpha + \beta + 2\gamma)/4$—a result which is always worse than $(5\alpha + 5\beta + 6\gamma)/16$. However, the question arises whether Nawaz and Toor's result is the best result which players can guarantee themselves in the quantum Battle of the Sexes game.

4. The Harsanyi–Selten algorithm of equilibrium selection

The algorithm of choosing equilibrium presented below is described in a renowned book by Nobel prize winners Harsanyi and Selten [10]. Its aim is to select in each 2×2 game with two strong equilibria only one of them or the equilibrium in mixed strategies.

To demonstrate an operation of the algorithm, let us consider the following 2×2 game:

$$\Delta : \begin{matrix} \\ p=1 \\ p=0 \end{matrix} \begin{matrix} q=1 & q=0 \\ \left[\begin{matrix} (a_{11}, b_{11}) \\ (a_{21}, b_{21}) \end{matrix}\right. & \left.\begin{matrix} (a_{12}, b_{12}) \\ (a_{22}, b_{22}) \end{matrix}\right] \end{matrix} \tag{9}$$

and denote by $u_1 = a_{11} - a_{21}$, $v_1 = a_{22} - a_{12}$, $u_2 = b_{11} - b_{12}$, $v_2 = b_{22} - b_{21}$. Furthermore, let us assume that the pairs of pure strategies $(1, 1)$, $(0, 0)$ form strong equilibria (an analogous criterion can be formulated for equilibria placed on the second diagonal). Then there exists also the third equilibrium (s_1, s_2) in mixed strategies, where $s_1 = v_2/(u_2 + v_2)$, $s_2 = v_1/(u_1 + v_1)$.

Algorithm 1. *From three equilibria the one which dominates according to payoffs, i.e. the one in which both players receive the largest payoffs, should be chosen. If this is not a case, then the equilibrium should be chosen according to the following formula:*

$$(r_1, r_2) = \begin{cases} (1, 1), & \text{if} \quad u_1u_2 > v_1v_2 \\ (0, 0), & \text{if} \quad u_1u_2 < v_1v_2 \\ (s_1, s_2), & \text{if} \quad u_1u_2 = v_1v_2. \end{cases} \tag{10}$$

Such a strategy pair is called a risk-dominant equilibrium [10].

It is important to note that the given algorithm is not contradictory to individual rationality. The algorithm should not be treated as an oracle which gives players unjustified hints which are in conflict with common sense. The criterion entirely reflects rational behavior of the players (see comments in [10]).

In order to see how this algorithm works, we apply it to the quantum version of the game the Battle of the Sexes studied by Nawaz and Toor in [9] and described by the payoff bi-matrix $\Gamma_{\rm NT}$ (8).

It can easily be noted that the game $\Gamma_{\rm NT}$ has three equilibria but none of them is dominant according to payoffs. Since $u_1 = u_2 = 4(\alpha - \gamma)$ and $v_1 = v_2 = 4(\beta - \gamma)$, we get $u_1u_2 = 16(\alpha - \gamma)^2 > 16(\beta - \gamma)^2 = v_1v_2$. Therefore, according to the rule given by the Harsanyi and Selten's algorithm, players should choose the equilibrium $(1, 1)$—a strategy pair which also Nawaz and Toor consider as the only rational solution in this game.

5. Dilemma of the Battle of the Sexes overcome

In the previous section, we presented the algorithm of equilibrium selection which should be adopted by rational players for 2×2 games with two strong equilibria. The quantum game begins when players receive the initial state and at this stage there is a need to define precisely its shape. The lemma below will allow one to use the Harsanyi–Selten algorithm and also to equalize players' preferences.

Let an initial state $|\psi_{\rm in}\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ of a quantum Battle of the Sexes game played according to the Marinatto–Weber scheme be given. Then the original classical game Γ (1) transforms into the game Γ' such that the following lemma holds:

Lemma 5.1. *If* $|a_{00}|^2 = |a_{11}|^2 = \frac{1}{2}(1 - (\epsilon_1 + \epsilon_2))$, $|a_{01}|^2 = \epsilon_1$, $|a_{10}|^2 = \epsilon_2$, *where* $\epsilon_1 + \epsilon_2 \leqslant 1 - 2\max\{\epsilon_1, \epsilon_2\}$ *for* $\epsilon_1 \neq \epsilon_2$ *and* $\epsilon < 1/4$ *for* $\epsilon_1 = \epsilon_2 = \epsilon$, *then for any real numbers* $\alpha > \beta > \gamma$:

(a) a game Γ' *is identical to* Γ *with respect to strategy profiles which constitute Nash equilibria in pure strategies and with respect to the number of equilibria,*
(b) payoff functions π'_A, π'_B *of the quantum game* Γ' *fulfil the condition:* $\pi'_A(r_1, r_2) = \pi'_B(r_1, r_2)$ *for all equilibria* (r_1, r_2) *of the game* Γ'.

Proof. Insert $|a_{00}|^2 = |a_{11}|^2 = \frac{1}{2}(1 - (\epsilon_1 + \epsilon_2))$, $|a_{01}|^2 = \epsilon_1$, $|a_{10}|^2 = \epsilon_2$ into formula (7). Taking into account assumptions of the lemma about the sum $\epsilon_1 + \epsilon_2$ we obtain

$$\begin{aligned} \pi'_A(1, 1) - \pi'_A(0, 1) &= \pi'_B(1, 1) - \pi'_B(1, 0) \\ &= (\alpha - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2\right) + (\beta - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1\right) > 0. \end{aligned} \tag{11}$$

Similarly,

$$\pi'_A(0,0) - \pi'_A(1,0) = \pi'_B(0,0) - \pi'_B(0,1)$$
$$= (\alpha - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1\right) + (\beta - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2\right) > 0. \quad (12)$$

□

We infer from this result that pairs (1, 1), (0, 0) form Nash equilibria, and none of the strategies is weakly dominated. Therefore, the game Γ' also possesses an equilibrium in mixed strategies.

Furthermore, it can easily be observed that $\pi'_A(1,1) = \pi'_B(1,1) = \pi'_A(0,0) = \pi'_B(0,0)$. Let us mark the third equilibrium of the game Γ' by (s_1, s_2). Due to $u_1 = u_2$ and $v_1 = v_2$, we obtain equality $(s_1, s_2) = (s_1, s_1) = (s_2, s_2)$. Therefore, besides the equalities $\pi'_A(1,0) = \pi'_B(0,1)$ and $\pi'_A(0,1) = \pi'_B(1,0)$, we get $\pi'_A(s_1,s_2) = \pi'_B(s_1,s_2)$.

The essential assumptions of the lemma are not *condito sine qua non* to fulfil the thesis. Taking into consideration, for example, another initial state: $|\psi_{\text{in}}\rangle = a_{01}|00\rangle + a_{00}|01\rangle + a_{11}|10\rangle + a_{10}|11\rangle$ one obtains a game which is identical to Γ' up to relabeling of strategies of one of the players. Moreover, the assumption $\epsilon_1 + \epsilon_2 \leqslant 1 - 2\max\{\epsilon_1, \epsilon_2\}$ can be weakened. The assumptions define the form of the initial state for the quantum Battle of the Sexes game with any $\alpha > \beta > \gamma$. The necessary and sufficient condition for inequalities (11) and (12) to be true requires dependence of ϵ_1 and ϵ_2 on α, β and γ. However, for simplifying the results, we will not go into details of this problem. As we will note further, the most important for our study are only the values of (ϵ_1, ϵ_2) in the neighborhood of (0, 0).

One of the characteristic features of both the classical game the Battle of the Sexes and any of its quantum versions is the lack of any equilibria which are dominating according to payoffs. However, the following theorem states that when assumptions of the lemma are fulfilled, then in the quantum version of this game a risk-dominant equilibrium exists.

Theorem 5.2. *If the quantum version Γ' of the game Γ fulfils assumptions of the lemma, then its risk-dominant equilibrium is the strategy profile:*

$$(r_1, r_2) = \begin{cases} (1,1), & \text{when } \epsilon_1 > \epsilon_2 \\ (0,0), & \text{when } \epsilon_1 < \epsilon_2 \\ (1/2, 1/2), & \text{when } \epsilon_1 = \epsilon_2. \end{cases} \quad (13)$$

Proof. Let us calculate u_1u_2 and v_1v_2 from the algorithm and estimate the difference, $u_1u_2 - v_1v_2$:

$$u_1u_2 = \left[(\alpha - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2\right) + (\beta - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1\right)\right]^2,$$
$$v_1v_2 = \left[(\alpha - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1\right) + (\beta - \gamma)\left(\tfrac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2\right)\right]^2,$$

consequently,

$$u_1u_2 - v_1v_2 = (\alpha + \beta - 2\gamma)(\alpha - \beta)(1 - 2(\epsilon_1 + \epsilon_2))(\epsilon_1 - \epsilon_2).$$

□

The first and second elements of the product are surely positive. Due to the assumption of the lemma, $1 - 2(\epsilon_1 + \epsilon_2)$ is also positive. Therefore, the sign of the difference, $u_1u_2 - v_1v_2$, depends only on the sign of the difference, $\epsilon_1 - \epsilon_2$.

In the case when $\epsilon_1 = \epsilon_2$, the game Γ' is characterized by the following equalities:

$$\pi'_A(r_1, r_2) = \pi'_B(r_1, r_2) \qquad \text{for all} \quad (r_1, r_2),$$
$$\pi'(i, j) = \pi'(i \oplus_2 1, j \oplus_2 1) \quad \text{for all} \quad i, j \in \{0, 1\}$$

which imply that Nash equilibrium in mixed strategies is formed by a pair of strategies (1/2, 1/2).

The initial state is known to the players; so according to the theorem it determines all the development of the game. The values of the payoff function corresponding to (13) are as follows:

$$\pi'_{A,B}(r_1, r_2) = \begin{cases} \frac{1}{2}\left[(\alpha+\beta) - (\alpha+\beta-2\gamma)(\epsilon_1+\epsilon_2)\right], & \text{when} \quad \epsilon_1 > \epsilon_2 \\ \frac{1}{2}\left[(\alpha+\beta) - (\alpha+\beta-2\gamma)(\epsilon_1+\epsilon_2)\right], & \text{when} \quad \epsilon_1 < \epsilon_2 \\ \frac{1}{4}(\alpha+\beta+2\gamma), & \text{when} \quad \epsilon_1 = \epsilon_2. \end{cases} \tag{14}$$

The payoff function depends only on the values of ϵ_1, ϵ_2; thus it can be identified with a function of two variables ϵ_1 and ϵ_2:

$$\pi'_{A,B}(\epsilon_1, \epsilon_2) = \begin{cases} \frac{1}{2}\left[(\alpha+\beta) - (\alpha+\beta-2\gamma)(\epsilon_1+\epsilon_2)\right], & \text{when} \quad \epsilon_1 \neq \epsilon_2 \\ \frac{1}{4}(\alpha+\beta+2\gamma), & \text{when} \quad \epsilon_1 = \epsilon_2. \end{cases} \tag{15}$$

This function is composed of two linear functions with variables ϵ_1, ϵ_2. Let us examine its limit:

$$\lim_{(\epsilon_1,\epsilon_2)\to(0,0)^+} \pi'_{A,B}(\epsilon_1, \epsilon_2) = \begin{cases} \frac{1}{2}(\alpha+\beta), & \text{when} \quad \epsilon_1 \neq \epsilon_2 \\ \frac{1}{4}(\alpha+\beta+2\gamma), & \text{when} \quad \epsilon_1 = \epsilon_2. \end{cases} \tag{16}$$

It follows that

$$\sup_{\epsilon_1,\epsilon_2} \pi'_{A,B}(\epsilon_1, \epsilon_2) = \tfrac{1}{2}(\alpha+\beta). \tag{17}$$

The maximum value of the function $\pi'_{A,B}(\epsilon_1, \epsilon_2)$ (15) does not exist, but for any small positive value δ an arbiter is able to prepare the initial state with sufficiently small ϵ_1, ϵ_2 that are different from each other in such a way that payoffs of players differ from $\frac{1}{2}(\alpha+\beta)$ by less than δ. This means that in the quantum Battle of the Sexes game both players may obtain equal payoffs arbitrary close to $\frac{1}{2}(\alpha+\beta)$.

Example 5.3. If $(\alpha, \beta, \gamma) = (5, 3, 1)$, then according to the result of Nawaz and Toor, each player gets payoff 2.875 while our formula yields for an initial state of the game characterized by $|a_{01}|^2 = \epsilon_1 = 0.01$, $|a_{10}|^2 = \epsilon_2 = 0.02$ and $|a_{00}|^2 = |a_{11}|^2 = 0.485$ payoffs $[(5+3) - 0.03(5+3-2)]/2 = 3.91$.

6. Conclusions

We obtained a new result in the quantum Battle of the Sexes game played according to the Marinatto–Weber scheme. In contrast to [6], we considered the initial state of the game to be the most general state of two qubits. We put conditions for amplitudes of the initial state so that the quantum form of the Battle of the Sexes game (1) has identical strategic positions of players as the initial game. In contrast to [9], we did not select a particular initial state, but we examined the dependence of players' payoffs on amplitudes of base states that form the initial state of the game. Our research showed that the initial state $|\psi_{\text{in}}\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ from [9] characterized by $|a_{00}|^2 = |a_{11}|^2 = |a_{01}|^2 = 5/16$, $|a_{10}|^2 = 1/16$ is one of many initial states that can be prepared without losing any characteristic feature of the classical Battle of the Sexes game. Moreover, we discovered infinitely more initial states for which players can achieve higher payoffs than by means of Nawaz and Toor's initial state. This allowed us to determine the supremum of the payoffs' values. This quantum version assures that its participants can get payoffs arbitrarily close to the equal for both players' maximal payoff possible in the game: $\frac{1}{2}(\alpha+\beta)$, which is the highest value that can be obtained in the 'classical' game if and only if players are allowed to communicate.

Acknowledgments

The author is very grateful to his supervisor Professor J Pykacz from the Institute of Mathematics, The University of Gdańsk, Poland for very useful discussions and a great help in putting this paper into its final form.

References

[1] Iqbal A Studies in the theory of quantum games arXiv:quant-ph/0503176
[2] Meyer D A 1999 Quantum strategies *Phys. Rev. Lett.* **82** 1052–5
[3] Eisert J and Wilkens M 2000 Quantum strategies *J. Mod. Opt.* **47** 2543
[4] Nawaz A and Toor A H 2006 Quantum games with correlated noise *J. Phys. A: Math. Gen.* **39** 9321–8
[5] Eisert J, Wilkens M and Lewenstein M 1999 Quantum games and quantum strategies *Phys. Rev. Lett.* **83** 3077–80
[6] Marinatto L and Weber T 2000 A quantum approach to static games of complete information *Phys. Lett.* A **272** 291–303
[7] Pykacz J and Frackiewicz P Arbiter as the third man in classical and quantum games arXiv:quant-ph/0707059
[8] Benjamin S C 2000 Comment on: a quantum approach to static games of complete information *Phys. Lett.* A **277** 180–2
[9] Nawaz A and Toor A H 2004 Dilemma and quantum battle of sexes *J. Phys. A: Math. Gen.* **37** 4437–43
[10] Harsanyi J and Selten R 1988 *A General Theory of Equilibrium Selection in Games* (Cambridge, MA: MIT Press)